Numeric Response Questions

Trigonometric Ratio

Q.1 If $\log \sin x - \log \cos x - \log 3(1 - \tan x) - \log (1 + \tan x) = -1$, then find the value of $\tan 2x$.

Q.2 If the value of $\cos \frac{2\pi}{15}, \cos \frac{4\pi}{15}, \cos \frac{8\pi}{15}, \cos \frac{14\pi}{15}$ is $\frac{1}{k}$ then find k.

Q.3 Find the maximum value of $12\sin \theta - 9\sin^2 \theta$.

Q.4 Find total number of solutions of $\sin x = \frac{|x|}{10}$.

Q.5 If $\sin \theta + \operatorname{cosec} \theta = 2$, then find the value of $\sin^2 \theta + \operatorname{cosec}^2 \theta$.

Q.6 If α is the root of $25\cos^2 \theta + 5\cos \theta - 12 = 0, \pi/2 < \alpha < \pi$, then find the value of $\sin 2\alpha$ is $\frac{-k}{25}$ then find k.

Q.7 If θ lies in the second quadrant and $3\tan \theta + 4 = 0$, then the value of $2\cot \theta - 5\cos \theta + \sin \theta$ is $\frac{6}{k}$ then find k.

Q.8 Find the value of expression $3\left[\sin^4 \left(\frac{\pi}{2} - \alpha\right) + \sin^{-1}(3\pi - \alpha)\right] - 2\left[\cos^6 \left(3\frac{\pi}{2} + \alpha\right) + \cos^6 (5\pi - \alpha)\right] + 6$.

Q.9 If $\tan (\alpha + \beta) = \frac{1}{2}$ and $\tan (\alpha - \beta) = \frac{1}{4}$, then find the value of $\tan (2a)$.

Q.10 If $\frac{\sin 3\theta + \sin 5\theta + \sin 7\theta + \sin 9\theta}{\cos 3\theta + \cos 5\theta + \cos 7\theta + \cos 9\theta} = \tan k\theta$ then find value of k.

Q.11 Find the value of $\frac{\cos 20° + 8\sin 70° \sin 50° \sin 10°}{\sin^2 80°}$,

Q.12 If the value of $\sin 20° \sin 40° \sin 60° \sin 80°$ is $\frac{3}{k}$ then find k.

Q.13 Find the value of $\cos 24° + \cos 5° + \cos 175° + \cos 204° + \cos 800°$.

Q.14 If $3\sin\ \theta + 5\cos\ \theta = 5$, then find the value of $5\sin\ \theta - 3\cos\ \theta$.

Q.15 Find the value of $\cot\ x \cdot \cot\ 2x - \cot\ 2x \cdot \cot\ 3x - \cot\ 3x . \cot\ x$.

ANSWER KEY

1. 0.67	**2.** 16.00	**3.** 4.00	**4.** 6.00	**5.** 2.00	**6.** 24.00	**7.** 2.30
8. 7.00	**9.** 0.86	**10.** 6.00	**11.** 2.00	**12.** 16.00	**13.** 0.50	**14.** 3.00

15. 1.00

Hints & Solutions

1. $\log_3 \tan x - \log_3 (1 - \tan^2 x) = -1$

$$\frac{\tan x}{1-\tan^2 x} = \frac{1}{3} \Rightarrow \tan 2x = \frac{2}{3}$$

2. $-\cos\frac{\pi}{5}\cos\frac{2\pi}{5}\cos\frac{4\pi}{5}\cos\frac{8\pi}{15}$

$$= -\frac{\sin\frac{16\pi}{15}}{2^4 \sin\frac{\pi}{5}} = \frac{1}{2^4}$$

3. $12\sin\theta - 9\sin^2\theta$

$= 4 - 4 + 12\sin\theta - 9\sin^2\theta$

$= 4 - (4 - 12\sin\theta + 9\sin^2\theta)$

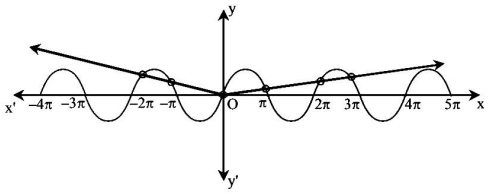
$= 4 - (2 - 3\sin\theta)^2$

Maximum value of given expression is 4

when $\sin\theta = \frac{2}{3}$

4. Let $y = \sin x$ ...(1)

$y = \frac{|x|}{10}$ ...(2)



six solution.

5. $\sin\theta + \frac{1}{\sin\theta} = 2$

so, $\sin\theta = 1$, $\text{cosec}\,\theta = 1$

Then $\sin^2\theta + \text{cosec}^2\theta = 1^2 + 1^2 = 2$

6. $25\cos^2\alpha + 5\cos\alpha - 12 = 0$

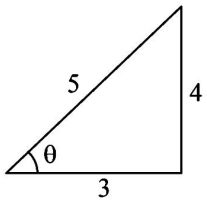
$\cos\alpha = \frac{-4}{5}$, $\cos\alpha = \frac{3}{5}$

(IInd quad.) (Not possible)

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$= \frac{-24}{25}$

7. $3\tan\theta + 4 = 0 \Rightarrow \tan\theta = \frac{-4}{3}$



$\cot\theta = -3/4$

$\cos\theta = -3/5$

$\sin\theta = 4/5$

$= 2\cot\theta - 5\cos\theta + \sin\theta$

$= 2(-3/4) - 5(-3/5) + 4/5 = \frac{23}{10}$

8. $3[\cos^4\alpha + \sin^4\alpha] - 2[\sin^6\alpha + \cos^6\alpha] + 6$

put $\alpha = 90^\circ$

$= 3 - 2 + 6 = 7$

9. $\tan(\alpha+\beta) = \frac{1}{2}$ and $\tan(\alpha-\beta) = \frac{1}{4}$

$\tan 2\alpha = \tan(\alpha+\beta+\alpha-\beta)$

$$\tan(2\alpha) = \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$\tan(2\alpha) = \frac{\frac{1}{2}+\frac{1}{4}}{1-\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)} = \frac{\frac{3}{4}}{\frac{7}{8}} = \frac{6}{7}$$

10. $$\frac{\sin 9\theta+\sin 7\theta+\sin 5\theta+\sin 3\theta}{\cos 9\theta+\cos 7\theta+\cos 5\theta+\cos 3\theta}$$

$$=\frac{2\sin 8\theta\cos\theta+2\sin 4\theta\cos\theta}{2\cos 8\theta\cos\theta+2\cos 4\theta\cos\theta}$$

$$=\frac{2\cos\theta(\sin 8\theta+\sin 4\theta)}{2\cos\theta(\cos 8\theta+\cos 4\theta)}$$

$$=\frac{2\sin 6\theta\cos 2\theta}{2\cos 6\theta\cos 2\theta}$$

$$=\tan 6\theta$$

11. $$\frac{\cos 20+8\sin 70.\sin 50\sin 10}{\sin^2 80}$$

$$=\frac{\cos 20+8\sin 10\sin(60-10)\sin(60+10)}{\sin^2 80}$$

$$\begin{cases}\because \sin A\sin(60-A)\sin(60+A)\\ =\sin 3A/4\end{cases}$$

$$=\frac{\cos 20+\frac{8\sin 30}{4}}{\sin^2 80}$$

$$=\frac{\frac{1+\cos 20}{1-\cos 160}}{2}$$

$$=\frac{2(1+\cos 20)}{1+\cos 20}=2$$

12. $\sin 20°\sin 40°\sin 60°\sin 80°$

$$=\frac{\sqrt{3}}{2}\sin 20°\sin(60°-20°)\sin(60°+20°)$$

$$\left[\sin\theta\sin(60°-\theta)\sin(60°+\theta)=\frac{\sin 3\theta}{4}\right]$$

$$=\frac{\sqrt{3}}{2}\frac{\sin(3\times 20°)}{4}$$

$$=\frac{\sqrt{3}}{2}\frac{\sin 60°}{4}$$

$$=\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{8}=\frac{3}{16}$$

13. $\cos 24° + \cos 5° + \cos(180° - 5°) + \cos(180° + 24°) + \cos(360° - 60°)$

$= \cos 24° + \cos 5° - \cos 5° - \cos 24° + \cos 60°$

$= 1/2$

14. $9\sin^2\theta + 25\cos^2\theta + 30\sin\theta\cos\theta = 25$ ...(i)

$25\sin^2\theta + 9\cos^2\theta - 30\sin\theta\cos\theta = x^2$ ...(ii)

$\Rightarrow 9 + 25 = x^2 + 25$

$\Rightarrow x = 3$

15. We know

$3x = 2x + x$

$\Rightarrow \cot 3x = \cot(2x + x)$

$$\Rightarrow \cot 3x = \frac{\cot 2x.\cot x - 1}{\cot 2x + \cot x}$$

$\Rightarrow \cot 3x\cot 2x + \cot 3x\cot x = \cot 2x\cot x - 1$

$\Rightarrow \cot x\cot 2x - \cot 2x\cot 3x - \cot 3x\cot x = 1$